

Vacuum Tunneling by Cosmic Strings

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Abstract

We consider vacuum tunneling of a new kind where the false vacua are not translationally invariant, but have topological defects that break some of their translational symmetries. In the particular case where the topological defects are cosmic strings, we show the existence of an $O(2) \times O(2)$ symmetric bounce configuration that allows a semi-classical estimate of the rate of cosmic string induced tunneling. A method of reduction is then suggested for simplifying the computation of the bounce action. Some phenomenological applications are described.

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1 Introduction

First order phase transitions in field theories may sometimes take place through the nucleation of bubbles seeded by massive excitations [1]. Of the several distinct processes that can lead to the seeding, a particularly interesting one is due to the instability of topological defects (like cosmic strings and monopoles) with respect to spontaneous dissociation. The phenomenon is best understood by an example. Consider the $SU(5)$ symmetric GUT theory. The sequence of phase transitions $SU(5) \rightarrow SU(4) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$ occurs as the universe cools from temperatures of 10^{15} GeV. At the end of the first phase transition $SU(4) \times U(1)$ monopoles are formed. These monopoles may have a structure such that at their cores the vacuum expectation values (VEV) of the scalar fields responsible for the second phase transition lie near the minimum of the effective potential of the $SU(3) \times SU(2) \times U(1)$ phase. As the temperature drops to the point where the $SU(4) \times U(1)$ phase is metastable compared to the $SU(3) \times SU(2) \times U(1)$ phase, the monopoles become unstable if it is energetically favourable for their cores to expand. In other words the monopoles themselves become bubbles of true vacuum inside a sea of false vacuum. It was shown in ref. [2] that this phenomenon may precede the nucleation of the usual bubbles in the false vacuum and the phase transition may take place essentially through bubbles seeded by monopoles.

An analogous situation is the case where the topological defects remain classically stable as the supercooling proceeds but may decay by quantum tunneling. The result is the same seeding effect discussed above. However seeded bubbles compete with unseeded bubbles on the same footing; both are quantum (or thermal) processes and suppressed due to the usual barrier penetration (or Boltzman) factors and one must compute the rate of nucleation of the two kinds of bubbles to decide whether the seeding has an appreciable effect or not.

The phenomenon is interesting in theories where the false vacuum is so long lived that the phase transition never takes place and the universe remains trapped in the supercooled phase forever. In the minimal supersymmetric model (MSSM), for instance, it may be possible that the true vacuum is charge and color breaking (CCB) [5] and the existing, viable vacuum is only a false vacuum. Recently it has been shown that such viable false vacua can exist in other low energy supersymmetry breaking models [6] that also permit the existence of $U(1)$ cosmic strings. In all these cases, the decay rate of the viable false vacuum provides interesting constraints on the model. However, the existence of topo-

logical defects in the false vacua of these theories is also quite generic and independent constraints can be obtained by considering the nucleation rate of seeded and unseeded bubbles. While the tunneling associated with unseeded bubbles is well understood semi-classically and the rate can be found by computing the action of $O(4)$ invariant bounce configurations of the underlying Euclidean field theory, a similar semiclassical treatment is needed in the case of the seeded bubbles for a useful estimate of the corresponding tunneling rates.

In this paper we treat a local region of space containing a topological defect as a translationally non-invariant (non-homogeneous) false vacuum which, classically, corresponds to a well defined local minimum of the energy functional of the theory. We show that there are new bounce configurations (or instantons) associated with these non-homogeneous vacua that permit a semiclassical understanding of the tunneling seeded by the defect. In particular there is a bounce with a large symmetry that appears as the global minimum of the action in an open space of field configurations. We then devise simple numerical methods to compute a lower bound on the corresponding semiclassical tunneling rates. Although the new bounce configurations are more complicated than the usual bounces, we use a method of “reduction” to simplify the computation to the point where it is not much harder than the $O(4)$ symmetric computations for the usual bounces [9].

We restrict ourselves to the zero temperature tunneling, since that is the most interesting case for theories where the false vacuum is viable and required to be very long lived. However, generalizing to the finite temperature case is straightforward and the computation of the free energy of the critical bubble is analogous to the computation of the bounce action in a 3 dimensional Euclidean field theory. Our treatment is also applicable to all types of topological defects except global texture (see [4] and references therein for a discussion of different types of defects), but for brevity we limit ourselves to the case of cosmic strings only. In this case the effect of the tunneling is the random appearance of bead-like bubbles along the length of the cosmic string, which captures the essential feature of tunneling seeded by extended defects. Note that cosmic strings also happen to be the most interesting type of defects from the point of view of cosmology ([8] and references therein).

The paper is organized as follows. In section 2 we prove the existence of the new type of bounce solutions and show that they appear as global minima of the action in an open space of field configurations. In section 3 we indicate some phenomenological applications and effects of the seeded tunneling phenomenon. Finally in section 4 we

suggest a method of “reduction” that makes the estimation of the bounce action feasible even in complicated field theory models.

2 The Bounce as a Global Minimum of Action

The semiclassical vacuum tunneling rate was computed in ref. [3] by a saddle point evaluation of the Euclidean path integral. In the case of a theory with a single real scalar field ϕ with a false vacuum at $\phi \equiv \phi_f$, the transition probability per unit volume per unit time was found to be:

$$\frac{\Gamma}{V} = \left(\frac{S_E[\bar{\phi}]}{2\pi\hbar} \right)^2 \exp \left(\frac{-S_E[\bar{\phi}]}{\hbar} \right) \left| \frac{\det'[-\partial^2 + U''(\bar{\phi})]}{\det[-\partial^2 + U''(\phi_f)]} \right|^{-1/2} \times (1 + O(\hbar)) , \quad (2.1)$$

where $S_E[\bar{\phi}]$ is the Euclidean action for the so called ‘bounce’ configuration $\bar{\phi}$ and $\partial^2 = \partial_\mu \partial_\mu$. The prime on the determinant indicates that the zero eigenvalues are omitted.

We will use the Abelian Higgs model with a single complex scalar Φ to make our arguments concrete. To develop the formalism we start with the Euclidean action:

$$S_E = \int d^4x \left[\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} |D\Phi|^2 + U(\Phi) \right] , \quad (2.2)$$

where $D_\mu \Phi = (\partial_\mu - iA_\mu)\Phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Let us suppose that the potential $U(\Phi)$ has a global minimum at Φ_t and a local minimum at $\Phi_f \neq 0$. The false vacuum at Φ_f may permit the existence of cosmic strings. A false vacuum with a single cosmic string of unit winding number is the lowest energy state with the following boundary condition for the VEV of the scalar field (in some appropriate gauge),

$$\lim_{x,y \rightarrow \infty} \Phi(r, \theta) = |\Phi_f| \exp(i\theta) \quad (2.3)$$

where $r^2 = x^2 + y^2$, $\tan(\theta) = y/x$. The cosmic string runs parallel to the z axis and is a static solution of the equations of motion. Apart from the cylindrical symmetry, the solution is invariant under translations in the z direction.

The translationally invariant and homogeneous false vacuum at $\Phi \equiv \Phi_f$ is metastable and its decay rate is determined by (2.1). The corresponding bounce configuration $\bar{\Phi}^0$ is likely to be an $O(4)$ symmetric field configuration and the tunneling rate corresponds to the nucleation rate of unseeded bubbles. However the lowest energy state with an infinite cosmic string is also a local minimum of the energy functional and one can estimate the

tunneling rate associated with its decay. We will treat a region of space with an isolated straight cosmic string as a classical false vacuum which is simply non-homogeneous. The formalism behind equation (2.1) applies. The only important difference is that the field configuration at the false vacuum is a space dependent function which is not invariant under translations in x and y directions. The bounce configuration in this case is likely to be more complicated and will certainly lack the $O(4)$ symmetry of the usual bounces. The corresponding tunneling rate will be interpreted as the nucleation rate of critical bubbles along the length of the cosmic string.

When the false vacuum is chosen to have a single infinite cosmic string with the above properties, the Euclidean action for the new bounces must be computed with the energy density of the false vacuum normalized to zero. Let us denote the field configuration corresponding to the local minimum of energy with a single cosmic string by Φ^0, A^0 and the corresponding covariant derivative and field strength by $D^0\Phi^0$ and $F_{\mu\nu}^0$. The appropriately normalized Euclidean action is then (dropping the subscript on S):

$$S = K_1 + K_2 + P + F_1 + F_2 + F_3, \quad (2.4)$$

with $K_1 = \int d^4x \frac{1}{2} [|D_z\Phi|^2 + |D_t\Phi|^2]$; $K_2 = \int d^4x \frac{1}{2} [|D_x\Phi|^2 + |D_y\Phi|^2 - |D_x\Phi^0|^2 - |D_y\Phi^0|^2]$; $P = \int d^4x [U(\Phi) - U(\Phi^0)]$; $F_1 = \int d^4x \frac{1}{4g^2} [F_{xy}^2 - (F_{xy}^0)^2]$; $F_2 = \int d^4x \frac{1}{4g^2} [F_{zx}^2 + F_{tx}^2 + F_{zy}^2 + F_{ty}^2]$ and $F_3 = \int d^4x \frac{1}{4g^2} [F_{zt}^2]$. Here we have used the result that, Φ^0 and A^0 have cylindrical symmetry around the z axis with $A_z^0 = A_t^0 = K_1^0 = F_{zx}^0 = F_{zy}^0 = F_{tx}^0 = F_{ty}^0 = 0$. We will refer to this solution of the equations of motion as the trivial solution G^0 . The regularized quantities K_2, F_1 and P are finite in the neighbourhood of G^0 .

The search for stationary points of the action is greatly simplified by imposing symmetries on the field configurations. We will impose rotational symmetries in the x, y as well as in the z, t plane. Introducing the polar coordinates $R^2 = z^2 + t^2$, $\tan(\gamma) = z/t$, $r^2 = x^2 + y^2$ and $\tan(\theta) = y/x$ we write $\Phi(x, y, z, t) = a(r, R)\exp(i\theta)$, $A_\theta = v(r, R)/r$ and $A_r = A_R = A_\gamma = 0$ where a and v are real. The action reduces to:

$$S = 2\pi^2 \int \int R dR r dr \left[\frac{1}{2} (|\partial_R a|^2 + |\partial_r a|^2) + \frac{1}{2} (1-v)^2 \frac{a^2}{r^2} + \frac{1}{4g^2 r^2} (\partial_r v)^2 + U(a) - \mathcal{L}_0 \right], \quad (2.5)$$

where the counterterm \mathcal{L}_0 is obtained by replacing a and v by the two functions $a_0(r)$ and $v_0(r)$ corresponding to the trivial solution G^0 . The above ansatz for the fields allows us to set $A_r, A_R, A_\gamma = 0$. Variations in Φ and A that do not preserve the ansatz contribute only positive definite terms to the action and therefore the ansatz minimizes the action

with respect to those variations (this can also be seen as a consequence of the principle of symmetric criticality [7]). Consequently a stationary point of the action in (2.5) is a stationary point of the full action (2.4). The ansatz also implies that $F_2 = F_3 = 0$. In the rest of the paper we always take $A_t, A_z, A_r, F_2, F_3 \equiv 0$ and enforce the $O(2) \times O(2)$ symmetry described above. Also, by fixing the phase of Φ in the above fashion we fix the gauge completely.

The trivial solution G^0 has the same $O(2) \times O(2)$ symmetry and is obtained by finding the minimum energy solution with the boundary conditions (2.3). The energy functional is given by

$$E = 2\pi \int dz \int r dr \left[\frac{1}{2} |\partial_r a|^2 + \frac{1}{2} (1-v)^2 \frac{a^2}{r^2} + \frac{1}{4g^2 r^2} (\partial_r v)^2 + U(a) \right]. \quad (2.6)$$

Minimizing E gives the equations of motion:

$$\begin{aligned} \frac{\partial^2 a}{\partial r^2} &= \frac{\partial U(a)}{\partial a} + (1-v)^2 \frac{a}{r^2} - \frac{1}{r} \frac{\partial a}{\partial r} \\ \frac{\partial^2 v}{\partial r^2} &= \frac{1}{r} \frac{\partial v}{\partial r} - 2g^2 a^2 (1-v). \end{aligned} \quad (2.7)$$

G^0 is the solution $a_0(r), v_0(r)$ of the above equations that satisfies the boundary conditions: $a_0(0) = 0, a_0(\infty) = |\Phi_f|, v_0(0) = 0, v_0(\infty) = 1$.

Note that with the normalization of (2.4) the action of G^0 is zero and it is a local minimum of S . In contrast, a bounce solution in this case must have the following properties:

- (i) It starts at Euclidean time $t = -\infty$ from the static cosmic string solution and ends at Euclidean time $t = +\infty$ at the same solution but is not given everywhere by the trivial time translation of the static string solution.
- (ii) The solution, which extremizes the action, is a saddle point of the action with a single direction of instability, i.e with a single negative eigenvalue of the Hessian $\frac{\partial^2 S}{\partial [\Phi, A]^2}$.
- (iii) All velocities ($d\Phi/dt$ and dA_μ/dt) are zero for a temporal slice at some finite time (which we can choose to be at $t = 0$). The field configuration at $t = 0$ is often called the turning point of the bounce.

The direction of instability associated with the bounce configuration is intimately related to scale transformations of the fields. For a systematic exploration of these transformations it will be useful to define a one parameter deformation of the action (2.4) in the following manner (after dropping F_2 and F_3):

$$S[\eta] = K_1[\eta] + K_2 + P + F_1; \quad (2.8)$$

where K_2, P, F_1 are as defined before, $K_1[\eta]$ is a deformation of K_1 defined by :

$$K_1[\eta] = \int d^4x \frac{1}{2} [|D_z \Phi|^{2-\eta} + |D_t \Phi|^{2-\eta}] . \quad (2.9)$$

In the limit $\eta \rightarrow 0$ we recover the usual Abelian Higgs model (with the constraint $A_z, A_r, F_2, F_3 \equiv 0$).

Under the scale transformation $(z, t) \rightarrow (\lambda z, \lambda t)$, with λ being a positive number, the terms in the deformed action $S[\eta]$ transform as: $K_1[\eta] \rightarrow \lambda^\eta K_1[\eta]$; $(K_2, P, F_1) \rightarrow \lambda^2(K_2, P, F_1)$. Under these transformations the configuration G^0 is left invariant while the finiteness of the action is preserved in the neighbourhood of G^0 . An extremum of the deformed action must be invariant under the scale transformation. Therefore at a stationary point of the deformed action we have,

$$\left. \frac{\delta S[\eta]}{\delta \lambda} \right|_{\lambda=1} = \eta (K_1[\eta]) + 2 [P + K_2 + F_1] = 0. \quad (2.10)$$

The second derivative of $S[\eta]$ at this point is:

$$\left. \frac{\delta^2 S[\eta]}{\delta \lambda^2} \right|_{\lambda=1} = \eta(\eta - 1)K_1[\eta] + 2 [P + K_2 + F_1] . \quad (2.11)$$

Since $K_1[\eta] > 0$ for any non-trivial field configuration, the above quantity is negative for a non-trivial extremum of $S[\eta]$ if $2 > \eta > 0$. We will define the quantity $V = [P + K_2 + F_1]$ and prove the following statement.

Statement 1: *A bounce configuration exists in the Abelian Higgs model with the deformed action provided $2 > \eta > 0$. The bounce is the global minimum of the action in an open space of field configurations.*

Proof: Let the space of all smooth field configurations having the boundary condition given by property (i) of the bounce, satisfying (2.10) and having the $O(2) \times O(2)$ symmetry $\Phi(x, y, z, t) = |\Phi|(r, R)\exp(i\theta)$ be C . The trivial solution G^0 lies in C . Every point in C automatically satisfies properties (i) and (iii) of the bounce. We need to show that there is a point in C that also satisfies property (ii).

We can split C into two parts $C = C_0 \oplus C_1$ where C_0 is a connected space in C containing G^0 . In fact C_0 consists of a single point G^0 since there exists a ball around G^0 such that all field configurations contained in the ball except G^0 have $V > 0$. This follows from observing that at the false vacuum the energy density is minimized and therefore G^0 is a local minimum of $S[\eta]$ with $V = 0$. However all nontrivial points on C have $K_1[\eta] > 0$ and therefore by (2.10) $V < 0$.

The space C_1 is not empty since one can always construct a non-trivial field configuration that satisfies (2.10) and the correct boundary conditions. By construction, the global minimum G in C_1 is an extremum of the action with respect to all allowed variations. By (2.11) it also has a single direction of instability corresponding to the scale transformations $(z, t) \rightarrow \lambda(z, t)$. Therefore it has property (ii) of the bounce. **Q.E.D.**

The limit $\eta = 0$ can be approached in the following way. By definition, G is the global minimum of C_1 . First we show that G remains a maximum with respect to variations in a direction “orthogonal” to C_1 even when $\eta = 0$. The easiest way to see this is to consider a curve $\mathcal{C}[p]$ with $p \in [-1, 1]$ in the space of all field configurations with the right boundary conditions and passing through G ($\mathcal{C}[0] = G$) in a direction “orthogonal” to the space C_1 . For $2 > \eta > 0$ one can draw this curve so that one end of the curve is at G^0 ($\mathcal{C}[-1] = G^0$) and the other end approaches the point G^1 where $\Phi \equiv \Phi_t$ ($\mathcal{C}[1] \rightarrow G^1$). Note that the space C_1 forms a boundary between the neighbourhoods of G^0 and G^1 . This is because $V[G^0] = 0$ and $V[G^1] \rightarrow -\infty$, therefore the curve $\mathcal{C}[p]$ must intersect the codimension one surface C_1 where $V < 0$. The curve can be drawn so that $S[\mathcal{C}[p]] \leq S[G]$ for all $p \neq 0$. As $\eta \rightarrow 0$, this property of the curve remains true and G remains a maximum of the action with respect to the variation that corresponds to moving along this curve. In this sense G remains a saddle point even when $\eta = 0$. In principle the negative eigenvalue of the Hessian $\frac{\partial^2 S}{\partial[\Phi, A]^2}$ may approach zero as $\eta \rightarrow 0$. Nevertheless, the zero eigenvalue can not be generic since there is no underlying symmetry that requires a zero eigenvalue. Phrased differently, all eigenvalues of the Hessian are likely to be of the order of m^2 where m is some characteristic mass scale in the theory. An eigenvalue can not be made zero without fine tuning the parameters of the theory, unless there is a symmetry to protect its smallness. Hence, for generic theories the limit $\eta \rightarrow 0$ can be safely taken.

Although the above discussion has been carried out in the context of an Abelian Higgs model with a single Higgs, everything said so far applies to the case of multiple Higgs fields. The generalization to the non-Abelian case is straightforward too; one simply needs to reduce the action in the non-Abelian case to an action resembling the Abelian model by dropping all components of the gauge field except one corresponding to a broken generator that generates a non-contractible loop on the vacuum manifold.

So far we have proven the existence of a bounce with a large symmetry ($O(2) \times O(2)$) that also happens to be the global minimum of the action in a simple space of field configurations. As we shall see, the latter property makes it quite simple to bound the action of this bounce from above. If this bounce is the bounce with the least action

then its action determines the tunneling rate of equation (2.1). If there is a bounce with a smaller action (and a lesser symmetry) then G provides only an upper bound on the least bounce action and consequently a lower bound on the tunneling rate. This bound itself may be useful in phenomenological applications and in section 4 we suggest simple numerical techniques for obtaining this bound.

3 Phenomenological Applications

The formal expression for the tunneling rate is similar to equation (2.1). However we must drop the zero modes from the primed determinant. There are just two zero modes of the bounce solution coming from translations in the z and t directions. To leading order in \hbar we have:

$$\gamma = \frac{\Gamma}{L} = \left(\frac{S[\bar{\Phi}]}{2\pi\hbar} \right) \exp \left(\frac{-S[\bar{\Phi}]}{\hbar} \right) \left| \frac{\det'[-\partial^2 + U''(\bar{\Phi}, \bar{A})]}{\det[-\partial^2 + U''(\Phi^0, A^0)]} \right|^{-1/2}, \quad (3.12)$$

where S is the action of the string induced bounce $[\bar{\Phi}, \bar{A}]$ (if and when it exists). Note that the configurations Φ^0, A^0 are space dependent. Corresponding to the two zero modes there are only two factors of $\left(\frac{S[\bar{\Phi}]}{2\pi\hbar} \right)^{1/2}$ on the R.H.S [3]. The ratio of the determinants in (3.12) now has the dimensions of mass squared and the transition rate is to be interpreted as the probability of transition per unit time *per unit length of the string*. Collecting all non-computable parts into a single prefactor \mathcal{A} we can rewrite the above equation as:

$$\gamma = \mathcal{A} \exp(-S[\bar{\Phi}, \bar{A}]/\hbar). \quad (3.13)$$

To a good approximation \mathcal{A} can be replaced by the characteristic squared mass scale of the theory and only the exponent needs to be determined carefully.

Phenomenologically the tunneling rate obtained from (3.13) is useful in two distinct situations. The first is the case when the theory has a viable false vacuum and one requires that the phase transition should never occur. The second case is a GUT model at high temperatures where one would like to calculate the high temperature vacuum decay rate and see if the string induced bubbles can percolate and complete the phase transition. We will briefly discuss both cases.

Consider the case of the viable false vacuum first. In this case, the false vacuum should have a lifetime longer than the age of the universe. A constraint is therefore obtained from the zero temperature tunneling rate. Multiplying the transition rate by the total

world sheet area of the cosmic strings, one gets the expected number of bubbles seeded by cosmic strings in a given space time. Most of the contribution to the string world sheet area comes from later times when the string network has entered a scale invariant distribution [8]. During these times about 80% of the string length is contained in one long string per horizon. The total world sheet area available for bubble nucleation is therefore $\sim H^2$ where H is the present horizon distance. The false vacuum would be regarded unstable if $\frac{\Gamma}{L} \times H^2 \geq 1$. If the mass scale of the theory is greater than the electroweak scale then this condition translates into the following bound on the bounce action:

$$S[\overline{\Phi}, \overline{A}] \leq 200 \hbar. \quad (3.14)$$

The same arguments repeated for unseeded bubbles yields the condition:

$$S[\overline{\Phi}^0, \overline{A}^0] \leq 400 \hbar. \quad (3.15)$$

Here $[\overline{\Phi}^0, \overline{A}^0]$ is the usual $O(4)$ invariant bounce and we have used the condition that $\frac{\Gamma}{V} \times H^4 \geq 1$. Notice that the tunneling rate is expressed as the number of bubbles per unit time per unit *volume* and correspondingly one uses the total space time *volume* available for nucleating the bubbles.

Cosmologically, there is no inflation in the false (but viable) vacuum and the bubbles, once formed, eventually percolate. The stability of the false vacuum therefore demands that both conditions (3.14) and (3.15) be untrue. The significance of this lies in the fact that a-priori the bounce actions $S[\overline{\Phi}, \overline{A}]$ and $S[\overline{\Phi}^0, \overline{A}^0]$ are not related in any simple manner and constraints obtained on the parameter space of the model by (3.14) are independent and may be stronger than the constraints implied by (3.15). The simplest way to see this is to note that it is possible to have theories where the cosmic strings are classically unstable [13]. By a continuous deformation of the parameters of a theory one can go from a regime of classically unstable strings to metastable strings. Clearly there is a continuous family of theories in between where the bounce action in (3.14) is arbitrarily small compared to the bounce action in (3.15) and the string induced tunneling is the dominant tunneling effect.

Now consider the case when the viable vacuum is the true vacuum. The interesting case arises when the universe gets trapped in the false vacuum and supercools. Then the phase transition must be completed by seeded and unseeded bubbles in an inflating universe which (in comoving coordinates) is described by the Robertson-Walker metric:

$$d\tau^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (3.16)$$

where $k = -1, 0$ or $+1$ depending on whether the universe is open, flat or closed respectively. The scale factor $R(t)$ grows according to the equation:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3M_p^2}\rho - \frac{k}{R^2}, \quad (3.17)$$

where $M_p = 1.2 \times 10^{19}$ GeV is the Planck mass. The energy density ρ is given by $\rho = \rho_0 + \left(\frac{\pi^2}{30}\right)\eta T^4$ where ρ_0 is the energy density of the false vacuum, η is the number of effectively massless degrees of freedom and T is the temperature. When the temperature is below the critical temperature T_c where the false vacuum becomes metastable, the energy density of the vacuum ρ_0 makes the dominant contribution to ρ . Approximating the energy density ρ by the vacuum energy density ρ_0 we obtain in the period of inflation: $\frac{\dot{R}}{R} \approx \chi\rho_0$, and the scale factor grows exponentially with time ($R \propto \exp(\chi t)$) where $\chi = \left(\frac{8\pi\rho_0}{3M_p^2}\right)^{1/2}$. With adiabatic expansion we also have: $RT = \text{constant}$, so the temperature falls exponentially with time.

In an inflating universe, the nucleation rate of unseeded bubbles grows as the volume of the universe ($\sim R^3$) while the same rate for seeded bubbles grows only as the length of the cosmic strings ($\sim R$). Nevertheless, the seeded bubbles can play a significant and even the dominant role in the phase transition. To see this consider the probability that an arbitrary point remains in the supercooled false vacuum at time t . This was found in ref. [11] to be:

$$p(t) = \exp\left[-\int_{t_c}^t dt' n(t')V(t, t')\right], \quad (3.18)$$

where $V(t, t') = \frac{4\pi}{3} \left[\int_{t'}^t \frac{dt''}{R(t'')}\right]^3$ is the coordinate volume at time t of a bubble formed at time t' and $n(t')dt'$ is the number of bubbles formed per unit coordinate volume between the times t' and $t' + dt'$ and t_c is the time corresponding to the temperature T_c . If the tunneling rates associated with seeded and unseeded bubbles are respectively γ_0 and γ respectively, we can write:

$$n(t) = \gamma_0(t)R^3(t) + \gamma(t)\sigma R(t)R^2(t_c), \quad (3.19)$$

where σ is the total string length per unit coordinate volume at time t_c .

Both γ_0 and γ are the finite temperature tunneling rates which depend on time (or temperature) and can be evaluated by computing the bounce action in the 3 dimensional Euclidean space using the finite temperature effective potential. The tunneling rates will however be greater or equal to their zero temperature values $\gamma(0)$ and $\gamma_0(0)$. With the

approximations made above we can re-write equation (3.18) as a function of temperature.

$$p(T) \leq \exp \left[-d \int_T^{T_c} \frac{dT_1}{T_1} \left(\frac{\gamma_0(0)}{T_1^3} + \frac{\sigma\gamma(0)}{T_1 T_c^2} \right) \left(\int_T^{T_1} dT_2 \right)^3 \right], \quad (3.20)$$

with $d = \frac{4\pi}{3\chi^4}$. One can look at the above expression in two limiting cases. If the seeded bubbles have little effect on the phase transition we can drop the terms multiplying γ from (3.20). Then in the limit $T \rightarrow 0$ we have:

$$p(T \rightarrow 0) \leq \exp \left[-d\gamma_0(0) \log \left(\frac{T_c}{T} \right) \right]. \quad (3.21)$$

If $d\gamma_0(0)$ is of the order of unity or greater, a fast phase transition is accomplished. For small values of $d\gamma_0(0)$ ($d\gamma_0(0) \leq 10^{-3}$ may be sufficiently small) there may be no percolation or the resulting universe may be too inhomogeneous if reheating is achieved through bubble collisions [12]. Therefore viability of a model requires that either the transition be fast with only a mild inflation or the first order phase transition is followed by a slow roll-over of the inflaton as in new inflation.

The effect of the seeded bubbles can be seen by putting $\gamma(0) = 0$ in (3.20). Then we get:

$$p(T \rightarrow 0) \leq \exp [-d\sigma\gamma(0)] . \quad (3.22)$$

In this case the fraction of unconverted space approaches a non-zero value ($p \neq 0$) as $T \rightarrow 0$. However if $d\sigma\gamma(0) \geq 1$ then the phase transition is fast and the resulting universe is homogeneous with the temperature at the end of the phase transition being of the order of T_c . Comparison of (3.21) and (3.22) shows that for $\sigma\gamma(0) \geq \gamma_0(0)$ the seeded bubbles make a significant contribution to the phase transition. The value of σ depends on the initial distribution of the cosmic strings. There are two extreme cases. If the temperature at which the strings are formed is close to T_c then the string network is at the friction dominated period when the inflation begins and $\sigma \sim T_c^2$. If on the other hand the strings are formed at a much higher temperature than T_c then they would have a scaling distribution at T_c and we would expect about one long string per horizon. This implies that $\sigma \sim T_c^4/M_{pl}^2$. The characteristic mass scale associated with the tunneling rates γ_0 and γ is likely to be T_c . Therefore we can write: $\gamma_0(0) \sim T_c^4 \exp(-S^0/\hbar)$ and $\gamma(0) \sim T_c^2 \exp(-S/\hbar)$, where S and S^0 are the corresponding zero temperature bounce actions. Using these expressions the condition for the seeded bubbles to be significant in the transition may be stated as: $S - \alpha \leq S^0$, where α lies in the interval $[0, \log(T_c^2/M_{pl}^2)]$. Given the fact that the two bounce actions are independent of each other, in any realistic model the above condition is as likely to be satisfied as not.

4 Computing The Bounce Action

In this section we present a simple technique for actually computing the the zero temperature tunneling rate. The basic idea is similar to the suggestions of ref. [9] and ref. [10]. Instead of looking for a saddle point of the action in the space of all configurations, one looks for the global minimum of the action in the reduced space C_1 . The restriction to the reduced space can be done by Lagrange multipliers. We may define an improved action \overline{S} by:

$$\overline{S} = S + \Sigma_i M_i. \quad (4.23)$$

The extra terms M_i added to the action are of the form $c_j B_j^n$ where c_j is a Lagrange multiplier, n is a positive number and B_j are functionals of the fields that vanish on the reduced space. For instance, $\overline{S} = S + c_1 V^2$ has local minima at the trivial solution G^0 as well as G . By choosing c_1 large one can essentially force an iterative minimization to take place along a path on the surface C_1 . Clearly the minimum one may obtain is either G or some other local minimum G' of S in C_1 . In either case one reaches a bounce. However, since C_1 is an infinite dimensional space, in practice one can not arrive at the point G or even a local minimum G' through finite methods. Indeed, the result of the search is likely to be a point which lies close to C_1 but may not even be on C_1 . Nevertheless, by performing a small scale transformation $(z, t) \rightarrow \lambda(z, t)$, one can make the final point of the search lie exactly on C_1 . Thus irrespective of the accuracy of the numerical method, the search always succeeds in finding an upper bound on the action of the bounce G which sets a useful lower bound on the tunneling rate.

Another advantage of this method is that it admits extreme simplifications. Since one can at best hope to find an upper bound on the action of G , the computation can be simplified enormously at an early stage by making the search space small. The only required element of the computation is that condition $V = 0$ be enforced so that the result of the search is a point on C_1 . We will use the example of the Abelian Higgs model to illustrate a simplified search of this kind where the problem will be reduced to solving an ordinary differential equation in a single variable. (As mentioned before, the non-Abelian case is reducible to the Abelian one). We start with the action given by (2.5). The most interesting case arises when the potential $U(\Phi)$ has a local minimum at $|\Phi| = \Phi_f \neq 0$ and a global minimum at $\Phi = 0$. This is, for instance, the case in the MSSM if the true vacuum is viable and the false vacuum breaks hypercharge. The cosmic strings in the false vacuum may be metastable or even classically unstable with respect to an expansion

of their cores. In the true vacuum, they simply do not exist.

The first step is to determine the functions $a_0(r)$ and $v_0(r)$ numerically by minimizing the energy functional (2.6) or by solving the equations (2.8) numerically. Next, consider the field configuration $a(r, R) = a_0(r - r'(R))$ for $r \geq r'(R)$; $a(r, R) = 0$ for $r < r'(R)$; $v(r, R) = v_0(r - r'(R))$ for $r \geq r'(R)$; $v(r, R) = 0$ for $r < r'(R)$; where $r'(R)$ is a real positive function of R satisfying $r'(R)|_{R \rightarrow \infty} = 0$ and $\frac{dr'}{dR}|_{R=0} = 0$. The field configuration consists of a string that nucleates a bubble of true vacuum in its “core” at some $t < 0$. The bubble expands as one approaches $t = 0$ and then it contracts and vanishes at some $t > 0$. For a given value of z and t , the cross section of the bubble in the x, y plane is a disc of radius r' .

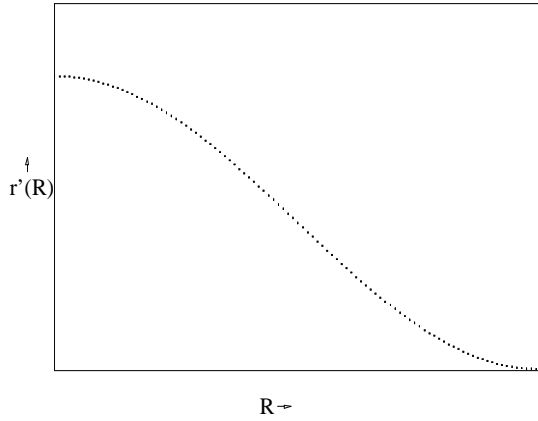


FIG. 1

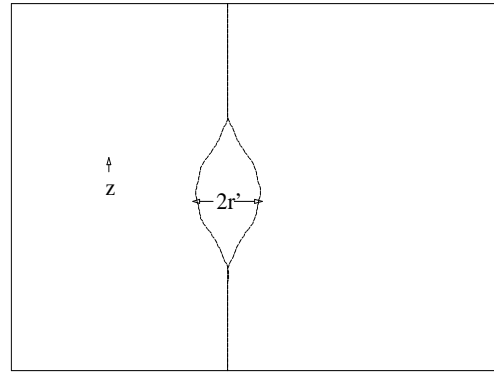


FIG. 2

FIG. 1. A typical form for the function $r'(R)$. The function goes to zero as $R \rightarrow \infty$.

FIG. 2. A schematic diagram for the string “core” after the nucleation of the bubble. The radius of the bubble in a plane perpendicular to the z axis is r' .

The above construction gives us a space of field configurations satisfying the correct boundary conditions and depending on a single real function r' of R . Restricted to this space of field configurations, the action is simply a functional of $r'(R)$ which can be thought of as the Lagrangian of a particle with coordinate r' integrated over time R . We have:

$$S = 2\pi^2 \int R dR \left[(L_1 + r' L_2) (\partial_R r')^2 + (L_3 + r' L_4) + \frac{U(0)}{4} r'^2 - \mathcal{L}_0 \right], \quad (4.24)$$

where $L = \left[\frac{a_0^2}{r^2} (1 - v_0)^2 + \frac{1}{2g^2 r^2} (\partial_r v_0)^2 \right]$; $L_1 = \int r dr (\partial_r a_0)^2$; $L_2 = \int dr (\partial_r a_0)^2$; $L_3 =$

$\int r dr L$; and $L_4 = \int dr L$. The integrals in r must be done numerically with the known functions a_0 and v_0 . Then the bounce in the reduced theory is obtained by solving the equation of motion implied by (4.24) which is an ordinary differential equation. This is exactly the same computation that one needs to do to evaluate the tunneling rate in the usual translationally invariant case for a theory with a single real scalar field. It can be easily done by the shooting method that has been described elsewhere [3, 9, 10].

Note that at the extremum, the action is extremized with respect to scale transformations of R which correspond to scale transformations in the z, t plane. Thus the extremum automatically lies on the surface C_1 and gives an upper bound on the action of G . In this case there is no further need to include Lagrange multiplier terms as in (4.23) to force the result to lie on the surface C_1 . This feature is typical of reductions of this kind where the final action depends on a single real function [9].

5 Conclusions

We have considered vacuum tunneling from non-homogeneous false vacua. A summary of our results is as follows:

(i) There are new tunneling phenomena associated with topological defects in a false vacuum. The rate of vacuum tunneling through bubbles seeded by defects is independent of (and can be arbitrarily large compared to) the tunneling rate through bubbles that are not seeded by the defects.

(ii) We have identified a bounce configuration (and call it G) which allows a semi-classical estimation of the tunneling rate.

(iii) It is possible to search for G numerically, by using simple minimization of a function on the lattice. We suggest a method of reduction by which one can do the search over a reduced space and find an upper bound on the action at G . The corresponding tunneling rate is a lower bound on the actual tunneling rate, and may be useful in placing constraints on the parameter space of realistic field theory models.

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